A Study on the Five-Axis End Milling for Sculptured Surfaces

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The direction vector of milling cutter for CL-data of five-axis milling is obtained by the fact that the bottom part of the milling cutter rides on free-form surfaces using the z-map method. Since the direction vector is known, CL-data can be transformed to the NC-code with regard to the geometry of the five-axis machine and post-processing. For uniform surfaces, the tool path is created from the prediction of cusp heights. After generating the NC-code, a sculptured surface was machined by five-axis end milling and cusp heights on the machined surface were measured by a three-dimensional CMM with laser scanner. From this machining test, it was found that this machining method is effective.

Key Words: Cutter Axis Direction, Cusp Height, Five-Axis Machining, Sculptured Surface, Tool Path

Nomenclature

P_{CC} : the position vector of the cc-point

X_G-Y_G-Z_G : global coordinate

X_T-Y_T-Z_T,X_C-Y_C-Z_C : local coordinate on work table and spindle

P_{XYZc},P_{XYZg} : the position vector of the bottom plane of the end mill cutter with respect to the X_C-Y_C-Z_C coordinate and X_G-Y_G-Z_G coordinate

P_C,P_T : the pivot point vector in the spindle and work table

A,B,C : swivel movements in NC-code, ^{o}

R,L_t : cutter radius, *mm*, and cutter length, *mm*

L_C : the pivot distance from the gage line to the pivot point in spindle, *mm*

L_T : the pivot distance from the work table to the pivot point in work table, *mm*

f, S : feedrate at cc-point, *mm/min*, and rotation speed of cutter, *rpm*

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1. Introduction

In three-axis machining of sculptured surface using a ball-end mill cutter, machinability at the bottom of the ball end mill cutter is poor. Also sometimes, workpieces having a complex geometry such as an impeller and an inclined hole can not be machined in three-axis milling. In addition to that, the ball end mill always produces the cusp on the machined surface. In order to decrease cusp heights in the machining of the sculptured surface with the ball end mill cutter, the tool path interval must be adjusted in consideration of the cusp height. Though this method can reduce polishing time, it requires extensive machining time. Even if a high speed machining method is used, since cutting conditions applying low cutting-force to spindle bearing of a high speed machine must be selected, cutting time by high speed machining process is greater than that by a traditional machining process. Therefore, both machining time and polishing time can not be controlled simultaneously at the present state of bearing technology. For these reasons, five-axis end milling has been recommended for an effective machining of the free-form surface (Tönshoff et al, 1989; Mason, 1991).

When machining sculptured surfaces on a five-axis CNC milling machine with the end mill cutter, the direction vector of the milling cutter must be determined inevitably (Vickers et al, 1989). The direction vector of the milling cutter is obtained by the fact that the bottom plane of the milling cutter must move along a tool path without interfering with free-form surfaces. Here, the z-map method is used for interference check. If the direction vectors are known, NC-code can be generated according to the geometry of five-axis milling machine and post-processing. In the machining of sculptured surfaces with five-axis milling machine and end mill cutter, cutter axis direction vectors become different with positions of the cutter contact point, and cusp heights are largely varied from this vector. For reference surfaces, tool path must be generated from the predicted cusp heights. If cusp height is obtained from the normal height between the surface and intersection point of two ellipses by the projection of the bottom plane of the milling cutter, these predicted cusp heights can be applied only to a straight tool path (Vickers et al, 1989). In this study, the cusp height is predicted from a mathematical modeling of cutting traces on the common plane defined along with the tool path. Since cusp heights in five-axis end milling are very small, grinding process may be omitted and only polishing process may be needed (Tönshoff et al, 1989). Thus, uniform surfaces are needed for the reduction of geometric error in this process. Also, straight tool path may be necessary for work convenience in the manual polishing process.

For machining experiments, a free-form surface was selected. This surface was machined by five-axis end milling with a 4mm straight tool path interval. Also the surface was machined with a curved tool path for uniform surface and straight tool path maintaining cusp height to a constant value. From test results and discussions, it was concluded that the machining of sculptured surfaces on the five-axis CNC machine was a very effective machining method.

2. The Method for 5-Axis End Milling

2.1 Coordinate representation

In most NC machines the work-table moves in the X and Y directions and the spindle moves in the Z direction. These are appropriately termed three-axis machines. Five-axis machine tools on the other hand, have two rotational freedoms in addition to the normal three orthogonal movements. These machines can be divided into three main families: Type 1. Machines with a fixed table and a spindle head capable of rotation in two perpendicular planes. Type 2. Machines with a fixed spindle and a table capable of rotation in two perpendicular planes. Type 3. Machines with a rotary table and a tilting spindle head.

The five-axis milling process is carried out at the cc-point as shown in Fig. 1, and coordinates can be represented for the cc-point. In all types of five-axis milling machines, the position of the NC command is the position (P_C or P_T) of the pivot point in the spindle head or the work-table. Since the workpiece is mounted on the work-table of the machine, the surface position of a part is represented by X_T-Y_T-Z_T local coordinate of table. If free-form surfaces are represented by

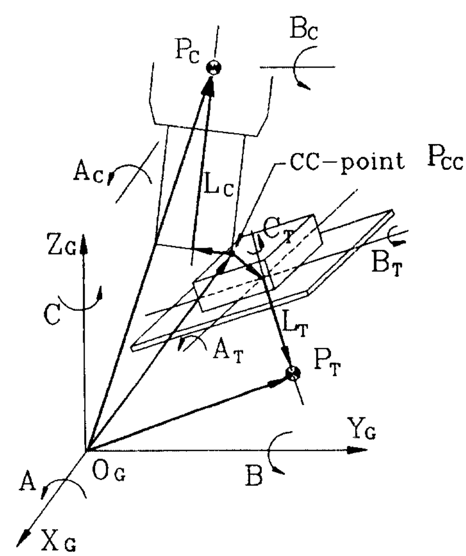
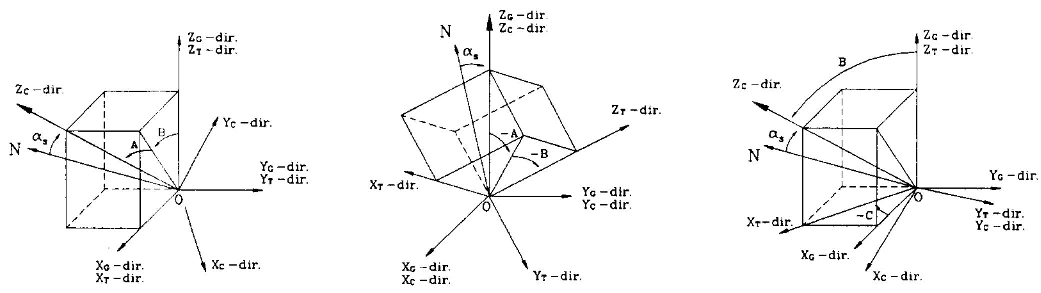


Fig. 1 Representation of coordinates in five-axis milling



(Type 1) (Type 2) (Type 3)

Fig. 2 Cutter axis direction vectors in three types of five-axis machines

parametric formulation, the surface position W_{XYZt} in a local coordinate of the table can be expressed in this way

$$W_{XYZt} = (W_{Xt}(u,v),\ W_{Yt}(u,v),\ W_{Zt}(u,v)) \quad (1)$$

If cutter axis direction vector $\mathbf{T}_a$ is known in types of five-axis machines, coordinates for cutter axis direction vector can be represented as shown in Fig. 2 (a)~(c). Where, $\mathbf{Z}_C$ direction is equal to cutter axis direction vector $\mathbf{T}_a$ and the center point O is cc-point represented by Eq. (1). If X_G-Y_G-Z_G global coordinate is equal to X_T-Y_T-Z_T local coordinate of a work-table in initial condition, the cc-point can be the position in global coordinate and $\mathbf{W}_{XYZt}$ can be translated to W_{XYZg}. In this study, coordinates are translated on the basis of cc-point. Here, this method is called the translation method on the basis of cc-point.

In the three types of five-axis machines, cutter axis direction vectors in the table coordinate system are represented respectively by

Type 1. $\quad \mathbf{Z}_{Ct} = \mathbf{M}_A(A)\mathbf{M}_B(B)\mathbf{Z}_G \quad (2)$

Type 2. $\quad \mathbf{Z}_{Ct} = \mathbf{M}_B(-B)\mathbf{M}_A(-A)\mathbf{Z}_G \quad (3)$

Type 3. $\quad \mathbf{Z}_{Ct} = \mathbf{M}_B(B)\mathbf{M}_C(-C)\mathbf{Z}_G \quad (4)$

where $\mathbf{Z}_{Ct}$ is the cutter axis direction vector in the local coordinate of a work-table centering on cc-point, $\mathbf{M}_A(A)$ is a translation matrix of the coordinate, and $\mathbf{Z}_G$ is $(0,0,1)^T$. Thus, if a cutter axis direction vector is known in the machining of sculptured surfaces with the end mill cutter, the A, B and C values can be calculated from Eq. (2), (3) and (4).

2.2 Cutter axis direction vector

In this study, the end mill cutter was used for the machining of a sculptured surface on a five-axis milling machine. In the previous section, the cutter axis direction vector required cl-data and NC-code machining sculptured surfaces on a five-axis milling machine used with the end mill cutter. The cutter axis direction vector had better be determined to produce minimum cusp heights on the machined surfaces at a fixed tool path interval.

In Fig. 3, X_p-Y_p-Z_p coordinate is the coordinate that the tool path direction projected on X_T-Y_T plane is equal to Y_p direction. α_{path} and α_{cpath} values are the angles determining the cutter axis direction vector in X_p-Y_p-Z_p coordinate. Since all positions of surface in a local coordinate can be obtained by translations, the z-values of Fig. 3 can be positions of surface in the X_p-Y_p-Z_p coordinate. From interference check by the extended z-map method, angular positions of α_{path} and α_{cpath} are obtained by adjustment of the milling cutter direction using the following procedure. As shown in Fig. 3, if the bottom plane of the milling cutter interferes with surfaces only at the left region, angle α_{path} is adjusted in a positive direction. Likewise, if the bottom plane interferes with surfaces only at the right region, angle α_{path} is adjusted in a negative direction. However, if the bottom plane interferes with surfaces at the left and right region simultaneously, angle α_{cpath} is adjusted in the negative direction. Therefore, the cutter axis direction vector can be obtained in the X_T-Y_T-Z_T coordinate.

2.3 Cusp height prediction

Generally, cusp heights in machining that util-

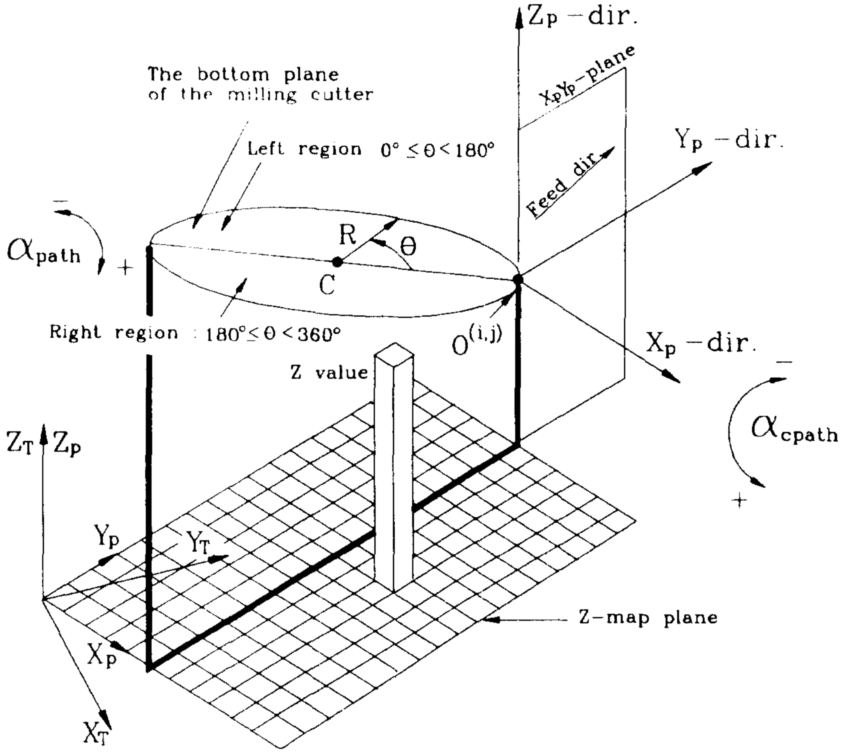


Fig. 3 Adjustment of the cutter axis direction using Z-map

izes the ball end mill cutter can be estimated explicitly, but the prediction of cusp heights in five-axis machining with the end mill cutter is very complex (Vickers et al, 1989). In this study, a common plane is defined for quantitative analysis of scallop, and a scallop on the common plane can be described from mathematical modelling. Thus, a cusp height can be obtained from the scallop on the common plane.

As shown in Fig. 4, the common plane is expressed as the plane that is formed by the vector connecting two cc-points between present tool path and next tool path and the summation vector of normal vectors at two cc-points. In Fig. 4, $\mathbf{N}$ is normal vector, $\mathbf{O}_t$ is the vector connecting two cc-points, $\mathbf{O}_m$ is the summation vector of two normal vectors, and $\mathbf{O}_n$ is the vector perpendicular to $\mathbf{O}_t$ on the common plane. If i notes a tool path and j notes a cc-point being in a tool path, the position vector of (i,j)-th cc-point is $\mathbf{P}_{CC}^{(i,j)}$. Also, the position vector $\mathbf{P}_{CO}^{(i,j)}$ of the common plane formed by (i,j)-th cc-point and (i+1,j)-th cc-point is expressed

$$\mathbf{P}_{CO}^{(i,j)}=\mathbf{O}^{(i,j)}+t_{p1}\,\mathbf{O}_t^{(i,j)}+t_{p2}\,\mathbf{O}_n^{(i,j)} \qquad (5)$$

where t_{p1} and t_{p2} are parameters, and vectors of Eq. (5) are represented in X_T-Y_T-Z_T local coordinate of the work table.

In any cc-point, the extended position vector of the bottom plane of an end mill cutter in X_C-Y_C-Z_C coordinate is expressed

$$\mathbf{P}_{XYZc}^{(i,j)}=[r\sin\theta\ r\cos\theta\ 0\ 1]^T \qquad (6)$$

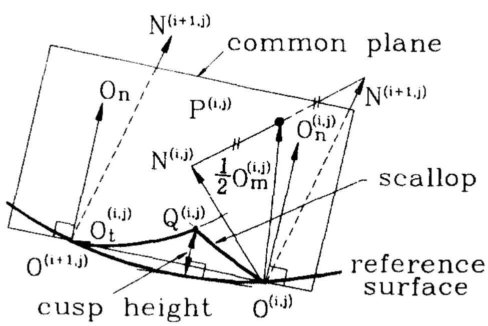


Fig. 4 Common plane and scallop

where r is from 0 to cutter radius R, and θ is from 0° to 360°. $P_{XYZc}^{(i,j)}$ can be converted to $P_{XYZt}^{(i,j)}$ by a coordinate translation. The machined surface is made by the motion of cutter. Therefore, the position vector of the end mill cutter changes along the tool path, the motion of the cutter.

The cusp heights on machined surface can be obtained from the trace of the bottom plane of the end mill cutter on the common plane. The trace can be obtained by following Eq. (7).

$$P_{CO}^{(i,j)} = P_{XYZt}^{(i,j+k)} \quad (7)$$

where index k, the infinite element of the tool path, is the range from start point 0 entering into the common plane to end point e getting out the common plane. If the position vector obtained from Eq. (7) is $C_{XYZt}^{(i,j+k)}(\theta)$ for a tool path passing through the common plane, the vector is transferred to $C_{tfn}^{(i,j+k)}(\theta)$ in the local coordinate of the common plane. Thus, the scallop at (i,j)-th cc-point is minimum value $(O_n^{(i,j)})$ of $C_{tfn}^{(i,j+k)}(\theta)$ vectors when the end mill cutter moves from its starting point to its ending point along i-th tool path. The scallop is described by

$$S_{tfn}^{(i,j)}(O_t^{(i,j)}) = \min(O_n^{(i,j)}) \prod_{k=0}^{k=e} (C_{tfn}^{(i,j+k)}(\theta))\, o_t^{(i,j)} \quad (8)$$

where $S_{tfn}^{(i,j)}(O_t^{(i,j)})$ is minimum value among $O_n^{(i,j)}$ components of $C_{tfn}^{(i,j+k)}(\theta)$ values when the k changes from 0 to e at the given $O_t^{(i,j)}$. Also, at (i+1,j) cc-point, the scallop is obtained such as Eq. (8). As a result, the scallop of the machined surface is the less value of both scallops at (i,j)-th and (i+1,j)-th tool paths. Therefore, the cusp height on the machined surface can be obtained from the scallop through the calculation process.

2.4 Tool path generation

In an NC machining of a die/mold, the determination of an effective tool path is important to the enhancement of precision and productivity. In this study, tool path is generated for precision machining. The cusp height predicted in previous section is important to determine tool path interval. Cusp heights on the sculptured surface machined by five-axis milling with the end mill cutter are much smaller than those by three-axis milling with the ball end mill cutter. Thus, in five-axis machining with the end mill cutter, the grinding process may be omitted and the machined part may be completed by the polishing process. Thus, a tool path interval had better be determined from cusp heights predicted in previous section. There are two kinds of tool paths. One is a curved tool path having uniform cusp height for polishing with spiral motion and the other is a straight tool path having less than given cusp height for polishing with straight motion.

2.5 Post processing

In the NC part program for the five-axis machine of Type 1 used in this study, the tool position (X,Y,Z) is the position of the pivot point, and the cutter direction (A,B) is expressed by the angle of the cutter axis rotating around the pivot point (Cincinnati Milacron Marketing Company, 1989).

Figure 5 shows the position of the pivot point. The position vector can be calculated from the summation of vectors and is expressed

$$P_v^{(i,j)} = O^{(i,j)} - R \cdot {}_uT_f^{(i,j)} + (L_t + L_p) \cdot {}_uT_a^{(i,j)} \quad (9)$$

Where, ${}_uT_f^{(i,j)}$ is the vector which center point vector (0, $-R\cos\alpha_{cpath}$, $-R\sin\alpha_{cpath}$) in X_p-X_p-Z_p coordinate is translated to X_G-Y_G-Z_G coordinate. Also, ${}_uT_a^{(i,j)}$ is the vector which Z_{Ct} in X_T-Y_T-Z_T coordinate is translated to X_G-Y_G-Z_G coordinate. Further, in the five-axis end milling of Type 2 and Type 3, translation of coordinates can be applied with rotational characteristics. Thus, considering

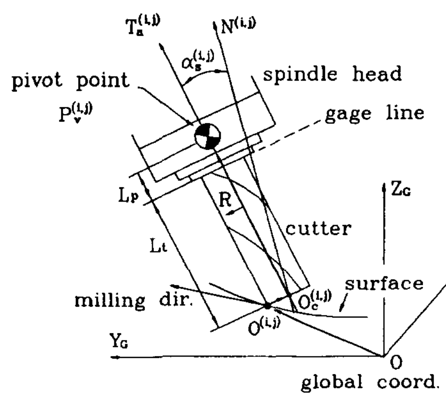


Fig. 5 The position of the pivot point

Fig. 1 and Eq. (2), (3) and (4), NC-code can be generated from the principle that coordinates are translated by the translation method on the basis of the cc-point.

Five-axis end milling produces the machined surface of edge shapes in feed direction and is termed the cutter mark. The magnitude of the edge height for five-axis milling is dependent upon rotation speed and feed. Thus, the cutter mark is expressed by

$$\varepsilon_v=\frac{f}{2n\cdot S}\sin(2\alpha_s^{(i,j)}) \tag{10}$$

where f is the feedrate at (i,j)-th cc-point, n is the number of cutter flute, $\alpha_s^{(i,j)}$ is 'Sturz' angle, and ε_v is the cutter mark along the feed direction as shown in Fig. 6. Thus, for fine surfaces, rotation speed or feedrate can be adjusted.

In the five-axis machining process by the NC-code with constant feedrate, machining speed may be very slow. This is due to the fact that both rotational and traverse movements are required simultaneously in programming (Cincinnati Milacron Marketing Company, 1989). This effect should be considered in five-axis post-processing. Therefore, for constant feedrate of the cutting edge at the cc-point, the feedrate of the pivot point must be varied along with the length of traversing and rotating path. Also, when machining from current position to command position with differential axis direction vector each other, an over-cut occurs by swivel movements around the pivot point. In order to solve this problem, the linearization of the tool path must be applied (Takeuch et al, 1992).

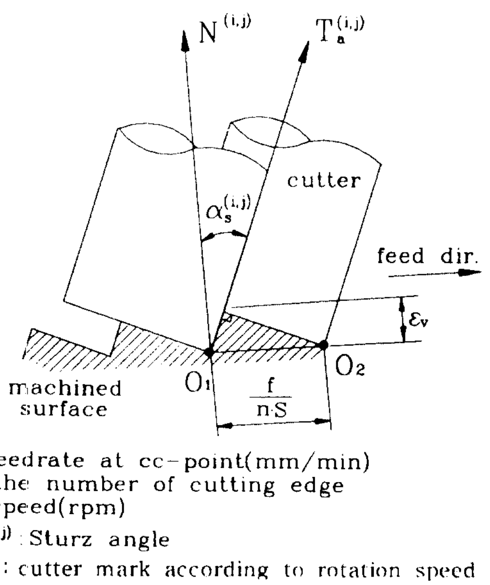


Fig. 6 The relation between feeding length and cutter mark according to one revolution of the end mill within $O^{(i,j)}O^{(i,j+1)}$

3. Experiments

The CINCINATI MILACRON five-axis machining center (model 20V-80) and three- dimensional coordinate measuring machine (CMM) (WEGU Messtechnik, model MMC 800) with laser scanner were used for tests in this study. This five-axis machine is included in the Type 1 group. For the machining experiment, a free-form surface was selected. The surface was machined by five-axis CNC milling with the end mill cutter. Bakelite, having a good machinability, was chosen for workpiece material. However, in machining tests for adjustment of cusp heights, aluminium was selected, since gloss of the machined surface showed the tool path clearly.

3.1 Experimental results and discussions

The NC-code was obtained from cutter axis direction vectors and five-axis post-processing. The selected free-form surface machined by five-axis end milling is shown in Fig. 7. Here, the radius of the end mill cutter is 8mm, feedrate is

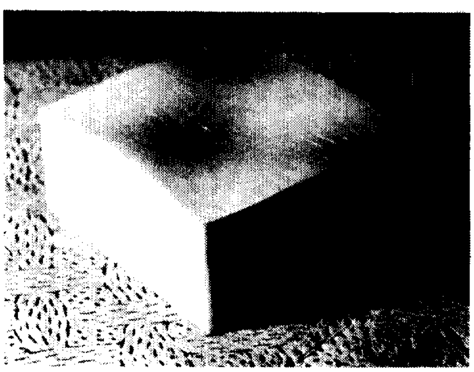


Fig. 7 Sculptured surface machined by five-axis end milling machine

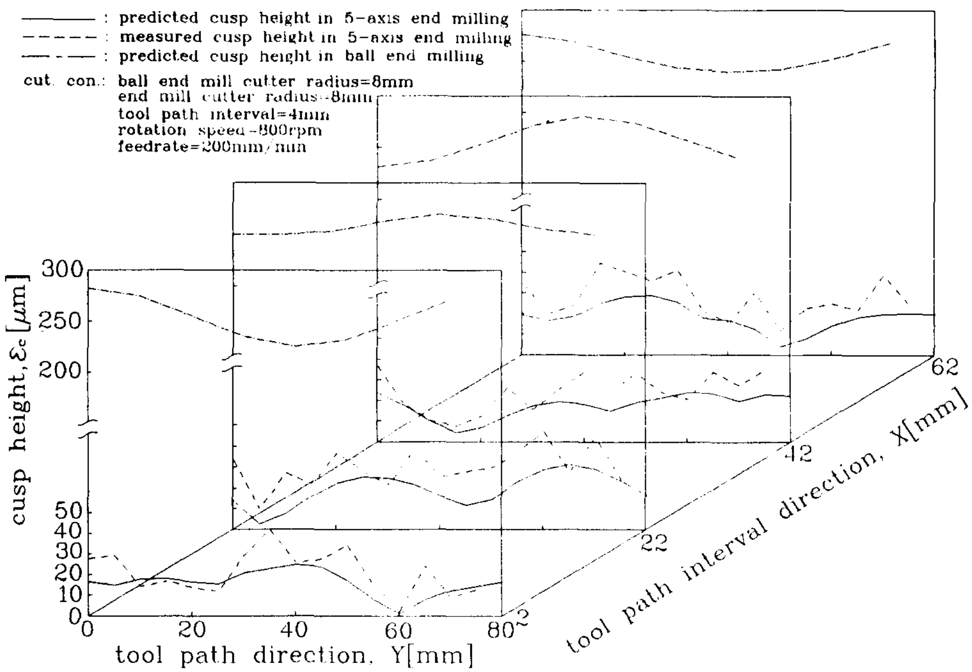
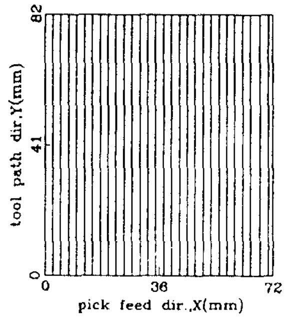


Fig. 8 Cusp heights on the surface of Fig. 7

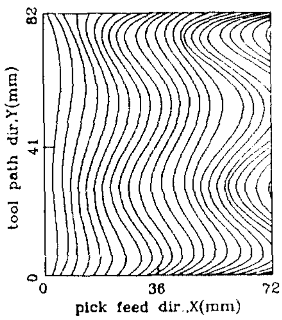
200mm/min at cc-point, the rotation speed of spindle is 800rpm, and tool path interval is 4mm.

The predicted and measured cusp heights on the machined surface are shown in Fig. 8. Since the prediction algorithm of cusp height was described very realistically, as shown in Fig. 8, predicted results harmonized well with test results. Thus, the prediction method of cusp height can be applied usefully for the determination of the tool path interval. In the machining of sculptured surfaces on a five-axis CNC machine with the end mill cutter, cusp heights vary in accordance with the geometric state of the surfaces, cutter radius, and tool path interval, etc. As shown in Fig. 8, the cusp heights on the mentioned sculptured surface machined by a five-axis CNC machining were about ten times smaller than those obtained by a three-axis ball end milling with the same cutting conditions. Therefore, polishing times for the surface machined by a five-axis end milling could be shortened significantly.

For a cartesian tool path in the three-axis machining of a sculptured surface with the ball end mill cutter, the variation of cusp heights is very small as compared to the cusp height. On the other hand, the variation in the five-axis end milling is very large. Thus, the surface machined by the three-axis ball end milling with the constant tool path interval has an almost uniform cusp height, but the surface machined by the proposed method of the five-axis end milling does not have a uniform cusp height. Therefore, if manual grinding and polishing processes are carried out following the three-axis ball end milling, the straight tool path may be advantageous. However, since the magnitude of the cusp height on the surface machined by the five-axis end milling is very small, the grinding process may be omitted and the machining industry may not required much labor for the polishing process. Since the sculptured surface of a final part is permitted small geometrical error and the machined surface having uniform cusp height can be polished with a spiral motion, the generation of surface having uniform cusp height is beneficial in five-axis end milling. Figure 9 shows the straight tool path constraining to constant cusp height and the curved tool path for uniform cusp height. Here, the surface model of Fig. 7 was used



(a) Straight cutter path



(b) Curved cutter path for uniform cusp height

Fig. 9 Straight cutter path having cusp heights under allowance value and curved cutter path maintaining cusp heights to allowance value

with a cutter radius of 8 mm, allowance cusp height of 10 μm, and initial tool path being straight line in the Y_T direction on X_T-Y_T plane. Figure 10 shows parts machined by tool paths of Fig. 9.

In the three-axis ball end milling and five-axis end milling of the sculptured surface, cutting times for uniform cusp height having allowance value, 10 μm, are shown in Table 1. Here, cutter diameters were selected ϕ8mm, ϕ10mm, ϕ12mm, ϕ16mm, and ϕ20mm used commercially in large numbers. Further, the feedrate at the cc-point was 200mm/min, and quick return speed was 5, 000mm/min. Since the surfaces machined by three-axis ball end milling and five-axis end milling have constant cusp heights relative to each other, their polishing times are similar. Thus, the result of Table 1 shows the fact that cutting times in five-axis end milling are shorter almost three times than those of three-axis ball end milling for uniform cusp height. Though the result is different along with the geometry of surfaces, machining times of sculptured surfaces by five-axis end milling are generally short. Finally, high speed machining is researched but is generally used in the machining of soft material such as aluminium alloy. This is due to the fact that a high speed spindle system which can endure large cutting force and maintain accuracy simultaneously for the machining of hard material, such as steel, has been not developed. Thus, high speed machining for steel is selected in order to reduce polishing times, but it leaves greater cutting times generally. From this viewpoint, we can propose that the machining method in this study has both the effect of high speed machining and shortens cutting times.

Also, as shown in Table 1, cutting times for three-axis ball end milling were shortened in large cutter radius, but cutting times for five-axis end

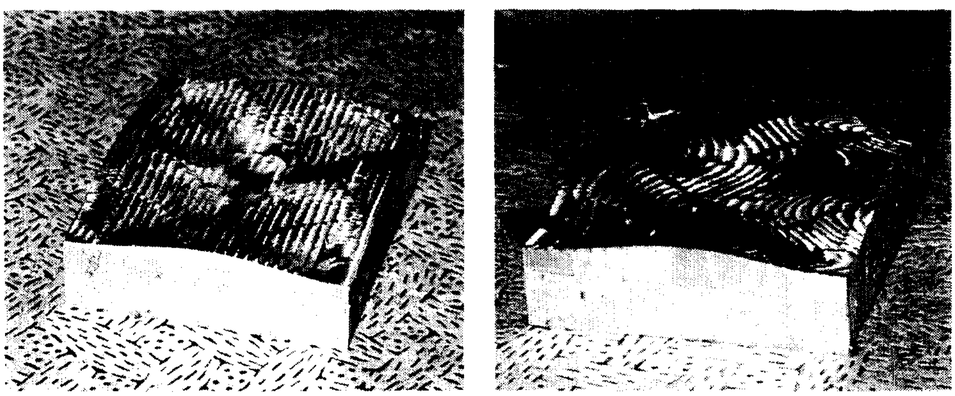


Fig. 10 Surfaces machined by cutter paths of Fig. 9

Table 1 Machining time along with cutter radius

cutter radius	time(min)		ratio
R(mm)	3-axis	5-axis	3-a/5-a
4	52.512	13.677	3.839
5	47.825	11.409	4.192
6	44.477	11.683	3.807
8	38.015	13.281	2.862
10	34.643	11.090	3.124

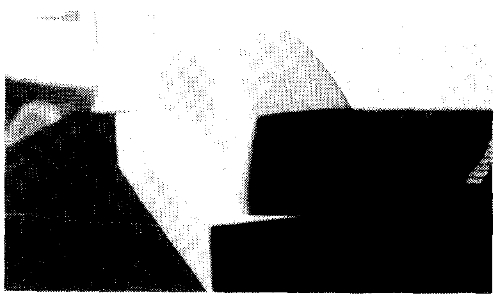


Fig. 11 Telephone receiver part machined by five-axis end milling

milling were not dependent upon this rule. Thus, in the three-axis ball end milling of sculptured surfaces, the largest ball end mill cutter that does not interfere with the surface is benefit. Then, the end mill cutter radius for an effective five-axis end milling of sculptured surfaces can be selected after the calculation of cutting times.

As shown in Fig. 11, from this study, a telephone receiver could be machined by five-axis end milling. This part consisted of many surfaces modelled individually, and was machined wholly by the end mill cutter, ϕ16mm. Cusp heights of the upper surface machined with 2mm tool path interval were nearly zero visionarily. This means that the machining of sculptured surfaces by the five-axis end milling is especially effective.

4. Conclusions

The machining of sculptured surfaces by five-axis end milling was very effective, and results of this study can be described as follows: On condition that the bottom plane of the end mill cutter does not interfere with surface at all, the direction vector of the milling cutter could be determined to produce minimum cusp height in the machining of the sculptured surfaces on a five-axis CNC machine. The five-axis NC-code could be generated effectively with the direction vectors of the milling cutter. The cusp height of the sculptured surface machined in the five-axis end milling was predicted, and the tool path was determined from the predicted cusp height. Machining times in five-axis end milling could be much smaller than those in three-axis ball-end milling.

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